Massive logarithmic graviton
in the critical generalized massive gravity

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Abstract

We study the generalized massive gravity in three dimensional flat spacetime. A massive logarithmic mode is propagating in the flat spacetime at the critical point where two masses degenerate. Furthermore, we discuss the logarithmic extension of the Galilean conformal algebra (GCA) which may arise from the exotic and standard rank-2 logarithmic conformal field theory (LCFT) on the boundary of AdS_3 spacetime.

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I. INTRODUCTION

Recently, a holographic correspondence between a conformal Chern-Simon gravity (CSG) in the flat spacetime and a chiral conformal field theory was reported in [1]. Choosing the CSG as the flat-spacetime limit of the topologically massive gravity in the scaling limits of $\mu \to 0$ and $G \to \infty$ while keeping fixed μG, two Bondi-Metzner-Sachs (BMS) central charges c_1 and c_2 are determined to be $c_1 = 24k$ and $c_2 = 0$. The CSG is conjectured to be dual to a chiral half of a CFT with $c = 24k$. This establishes the BMS/GCA correspondence [2–4] when choosing a concrete model of the CSG.

The linearized equation of the CSG leads to the third order equation $D^3 h = 0$ in the Minkowski spacetime. The solution to the first-order massless equation $Dh^\xi = 0$ is given by $h^\xi_{\mu\nu} = e^{-i(\xi+2)\theta} r^{-(\xi+2)} (m_1 \bigotimes m_1)_{\mu\nu}$ in Ref. [1], where ξ is the eigenvalue of GCA generator L_0 and m_1 is an ISO $(2,1)$ generator $m_1 = ie^{i\theta}(\partial_u - \partial_r - i\partial_\theta/r)$. Furthermore, the solution to $D^2 h^{\log} = 0$ is given by $h^{\log}_{\mu\nu} = -i(u+r)h^\xi_{\mu\nu}$, while the solution to $D^3 h^{\log^2} = 0$ is $h^{\log^2}_{\mu\nu} = -\frac{1}{2}(u+r)^2 h^\xi_{\mu\nu}$. Even though $\{h^{\log}, h^{\log^2}\}$ do not include logarithmic terms, they were considered as the flat-spacetime analogues of log- and $\log^2$-solutions on the AdS$_3$ spacetime. We would like to mention that the solutions $\{h^\xi, h^{\log}, h^{\log^2}\}$ could not represent any physical modes propagating on the flat spacetime background because the CSG has no local degrees of freedom. Actually, these all belong to the gauge degrees of freedom.

Hence, it is crucial to find a relevant gravity theory which has a physically massive mode propagating on the Minkowski spacetime. One model is given by the topologically new massive gravity (TNMG) [5, 6]. Very recently, we have shown that the linearized TNMG provides a single spin-2 mode with mass m^2/μ in the Minkowski spacetime [7]. On the other hand, it is well known that the cosmological generalized massive gravity (cGMG) has two different massive modes [8, 9]. Furthermore, the cGMG provides the standard rank-2 LCFT for $m_1\ell = 1$, while it yields the exotic rank-2 LCFT for $m_1\ell = m_2\ell \neq 1$ [10].

In this work, we explore the critical GMG where two masses degenerate in the Minkowski spacetime. We obtain the massive logarithmic wave solution and discuss the logarithmic extension of the GCA which may arise from the exotic rank-2 LCFT on the boundary of the AdS$_3$ spacetime. Furthermore, we show that in the flat-spacetime limit of $\ell \to \infty$, the cGMG for $m_1\ell = 1$ reduces to a logarithmic GCA in the semi-infinite spacetime with a boundary condition at $\phi = 0$, while the critical GMG yields null in the limit of $\ell \to \infty$.

In other words, the standard rank-2 LCFT survives in the flat-spacetime limit, whereas the exotic rank-2 LCFT disappears in the flat-spacetime limit.

II.   GMG AND ITS CRITICAL THEORY

The GMG action is given by [8, 9]

$$I_{\mathrm{GMG}} = \frac{1}{\kappa^2} \int d^3x \sqrt{-g}(\sigma R) + I_{\mathrm{CSG}} + I_{\mathrm{K}}, \tag{1}$$

where

$$I_{\mathrm{CSG}} = \frac{1}{2\kappa^2 \mu} \int d^3x \sqrt{-g} \epsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left(\partial_\mu \Gamma^\sigma_{\rho\nu} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right),$$

$$I_{\mathrm{K}} = \frac{1}{\kappa^2 m^2} \int d^3x \sqrt{-g} \left(R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right) \equiv \frac{1}{\kappa^2 m^2} \int d^3x \sqrt{-g} K. \tag{2}$$

Here $\kappa^2 = 16\pi G$ is the three-dimensional gravitational coupling, μ the Chern-Simons coupling, and m^2 the mass parameter for a higher curvature combination K. σ is chosen to be 1 for our purpose. The Einstein equation of the GMG action is given by

$$\sigma G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{1}{2m^2} K_{\mu\nu} = 0, \tag{3}$$

where the Einstein tensor $G_{\mu\nu}$ is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{4}$$

the Cotton tensor $C_{\mu\nu}$ takes the form

$$C_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \nabla_\alpha \left(R_{\beta\nu} - \frac{1}{4} g_{\beta\nu} R \right), \tag{5}$$

and the tensor $K_{\mu\nu}$ is given by

$$K_{\mu\nu} = 2\nabla^2 R_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R - \frac{1}{2} \nabla^2 R g_{\mu\nu} + 4 R_{\mu\rho\nu\sigma} R^{\rho\sigma}$$
$$- \frac{3}{2} R R_{\mu\nu} - g_{\mu\nu} R_{\rho\sigma} R^{\rho\sigma} + \frac{3}{8} R^2 g_{\mu\nu}. \tag{6}$$

As a solution to the Einstein equation (3), the Minkowski spacetime is chosen to be

$$ds^2_{\mathrm{EF}} = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -du^2 - 2drdu + r^2 d\theta^2, \tag{7}$$

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where $u = t - r$ is a retarded time expressed in terms of the outgoing Eddington-Finkelstein (EF) coordinates. In this spacetime, non-vanishing Christoffel symbols are given by

$$\Gamma^u_{\theta\theta} = r, \quad \Gamma^r_{\theta\theta} = -r, \quad \Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r}, \tag{8}$$

while all the components of curvature tensor $R_{\mu\nu\rho\sigma}$ are zero.

Considering the perturbation $h_{\mu\nu}$ around the EF background $\bar{g}_{\mu\nu}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{9}$$

the linearized equation of Eq. (3) takes the form

$$\sigma \delta G_{\mu\nu}(h) + \frac{1}{\mu}\delta C_{\mu\nu}(h) + \frac{1}{2m^2}\delta K_{\mu\nu}(h) = 0, \tag{10}$$

where linearized tensors are given in [11] with $\Lambda = 0$. It is necessary to choose the transverse and traceless gauge conditions to select two massive modes propagating on the EF background as

$$\bar{\nabla}^\mu h_{\mu\nu} = 0, \quad h^\mu{}_\mu = 0. \tag{11}$$

Then, we have the linearized fourth-order equation of motion as

$$\bar{\nabla}^2 \left[\bar{\nabla}^2 h_{\mu\nu} + \frac{m^2}{\mu}\epsilon_\mu{}^{\alpha\beta}\bar{\nabla}_\alpha h_{\beta\nu} + \sigma m^2 h_{\mu\nu} \right] = 0. \tag{12}$$

Introducing three mutually commuting operators as

$$D_\mu^\beta = \epsilon_\mu{}^{\alpha\beta}\bar{\nabla}_\alpha, \quad (D^{m_i})_\mu^\beta = \delta_\mu^\beta + \frac{1}{m_i}\epsilon_\mu{}^{\alpha\beta}\bar{\nabla}_\alpha, \quad i = 1, 2, \tag{13}$$

the fourth order equation (12) can be expressed

$$\left(D^2 D^{m_1} D^{m_2} h \right)_{\mu\nu} = 0, \tag{14}$$

where two masses are defined by

$$m_1 = \frac{m^2}{2\mu} + \sqrt{\frac{m^4}{4\mu^2} - \sigma m^2}, \quad m_2 = \frac{m^2}{2\mu} - \sqrt{\frac{m^4}{4\mu^2} - \sigma m^2}. \tag{15}$$

We note that D is the massless operator, while D^{m_i} correspond to two massive operators. Two masses are positive for $m^2 > 4\mu^2$ with $\sigma = 1$. For $m^2 = 4\mu^2$, two masses degenerate as

$$m_1 = m_2 = 2\mu \equiv m_0, \tag{16}$$

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which leads to a critical GMG in the flat spacetime.

In order to obtain the wave solution to the critical GMG, we first have to solve the first-order massive equation

$$(D^{m_i}h)_{\mu\nu} = h_{\mu\nu} + \frac{1}{m_i}\epsilon_\mu{}^{\alpha\beta}\bar\nabla_\alpha h_{\beta\nu} = 0 \tag{17}$$

under the traceless and transverse gauge conditions (11). Since the massive solution was already found for the case of single degree of freedom [7], we could write down the solution for the case of double degrees of freedom as follows

$$h_{\mu\nu}^{m_i}(u,r,\theta) = e^{-im_i(u+r)}e^{-2i\theta}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -ir \\ 0 & -ir & -r^2 \end{pmatrix} \equiv e^{-im_i(u+r)}e^{-4i\theta}(m_1\bigotimes m_1)_{\mu\nu}. \tag{18}$$

Then, at the critical point, the Einstein equation has reduced to

$$\left[(D^{2\mu})^2 h^c\right]_{\mu\nu} = 0. \tag{19}$$

Its solution is found to be

$$h_{\mu\nu}^c = \partial_{m_i} h_{\mu\nu}^{m_i}|_{m_i\to 2\mu} = -i(u+r)h_{\mu\nu}^{2\mu}, \tag{20}$$

which is our main result. Here we observe that there is no log-term in the critical tensor wave solution $h_{\mu\nu}^c$ in the flat spacetime, in comparison to the logarithmic solution $\psi_{\mu\nu}^{\log} = -[i\tau + \ln\cosh\rho]\psi_{\mu\nu}^{L}$ on the AdS$_3$ spacetime.

III. RE-DERIVING THE CRITICAL WAVE SOLUTION

It is important to justify the critical wave solution (20) because it was newly derived in the flat spacetime. In order to re-derive (20), we first consider the AdS/CFT correspondence on the AdS$_3$ and its boundary. The global AdS$_3$ spacetime could be described by the line element

$$ds_{\text{AdS}_3}^2 = \ell^2\Big(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\phi^2\Big), \tag{21}$$

where all coordinates are defined to be dimensionless.

Two central charges of the cGMG on the boundary are given by [11–13],

$$c_L = \frac{3\ell}{2G}\Big(\sigma + \frac{1}{2m^2\ell^2} - \frac{1}{\mu\ell}\Big), \quad c_R = \frac{3\ell}{2G}\Big(\sigma + \frac{1}{2m^2\ell^2} + \frac{1}{\mu\ell}\Big). \tag{22}$$

To obtain the GMG in the flat limit of $\ell \to \infty$, the corresponding BMS central charges are defined to be

$$c_1 = \lim_{\ell \to \infty} (c_R - c_L) = \frac{3}{G\mu}, \quad c_2 = \lim_{\ell \to \infty} \frac{c_R + c_L}{\ell} = \frac{3\sigma}{G}, \tag{23}$$

which show the absence of the mass parameter m^2 in the flat spacetime limit. We introduce the scaling limits of $\mu \to 0$ and $G \to \infty$ while keeping fixed μG. Considering a relation $G\mu = 1/8k$, its dual CFT is given by the 2D GCA with central charges [1]

$$c_1 = 24k, \quad c_2 = 0. \tag{24}$$

Considering the highest weight condition of the cGMG on the AdS$_3$ spacetime: $\mathcal{L}_0|\psi_{\mu\nu}> = h|\psi_{\mu\nu}>$ and $\bar{\mathcal{L}}_0|\psi_{\mu\nu}> = \bar{h}|\psi_{\mu\nu}>$, the connection between the GCA and the Virasoro algebras is given by

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \frac{\bar{\mathcal{L}}_n + \mathcal{L}_{-n}}{\ell}. \tag{25}$$

Here h and $\bar{h}$ are given for the cGMG by [10]

$$(h_{1/2}, \bar{h}_{1/2}) = \left(\frac{3 + \ell\tilde{m}_{1/2}}{2}, \frac{-1 + \ell\tilde{m}_{1/2}}{2} \right) \tag{26}$$

where

$$\tilde{m}_{1/2} = \frac{m^2}{2\mu} \pm \sqrt{\frac{1}{2\ell^2} + \frac{m^4}{4\mu^2} - \sigma m^2}. \tag{27}$$

Since the rigidity ξ and scaling dimension Δ are eigenvalues as

$$L_0|\Delta, \xi> = \xi|\Delta, \xi>, \quad M_0|\Delta, \xi> = \Delta|\Delta, \xi>, \tag{28}$$

they are given by

$$\xi = \lim_{\ell \to \infty} (h - \bar{h}), \quad \Delta = \lim_{\ell \to \infty} \frac{h + \bar{h}}{\ell}. \tag{29}$$

We obtain

$$\xi = 2, \quad \Delta = m_i, \tag{30}$$

where m_i is already given by Eq. (15). The eigenvalue $\xi = 2$ arises because it represents spin-2 excitations.

Now, it is very interesting to know what form of the GMG [11] provides Eq. (20) in the flat-spacetime limit. For the condition of

$$\frac{1}{2\ell^2} + \frac{m^4}{4\mu^2} - \sigma m^2 = 0, \tag{31}$$

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one has the degenerate mass from Eq. (27)

$$\tilde{m}_0 = \frac{m^2}{2\mu},$$

(32)

which provides the exotic rank-2 LCFT on the boundary of the AdS$_3$ spacetime [10]. In this case, the critical GMG wave solution in the light-cone coordinate is described by

$$\psi_{\mu\nu}^{\mathrm{elog}}(\rho, \tau^+, \tau^-) = \partial_{\tilde{m}_i} \psi_{\mu\nu}^{\tilde{m}_i}|_{\tilde{m}_i \to \tilde{m}_0} = y(\tau, \rho)\psi_{\mu\nu}^{\tilde{m}_0}(\rho, \tau^+, \tau^-)$$

(33)

where the logarithmic function y is

$$y(\tau, \rho) = -[i\tau + \ln \cosh \rho]\ell$$

(34)

and the massive wave function is

$$\psi_{\mu\nu}^{\tilde{m}_0}(\rho, \tau^+, \tau^-) = f(\rho, \tau^+, \tau^-) \begin{pmatrix} 1 & 0 & \frac{2i}{\sinh(2\rho)} \\ 0 & 0 & 0 \\ \frac{2i}{\sinh(2\rho)} & 0 & -\frac{4}{\sinh^2(2\rho)} \end{pmatrix}$$

(35)

with $\tau^\pm = \tau \pm \phi$. Here

$$f(\rho, \tau^+, \tau^-) = e^{-ih(\tilde{m}_0)\tau^+ - i\bar{h}(\tilde{m}_0)\tau^-}(\cosh \rho)^{-[h(\tilde{m}_0)+\bar{h}(\tilde{m}_0)]} \sinh^2 \rho.$$

(36)

We express the global coordinates (21) in terms of the EF coordinates (7) in Ref. [1]

$$u = \ell(\tau - \rho), \quad r = \ell\rho, \quad \theta = \phi.$$

(37)

Taking the limit of $\ell \to \infty$ while keeping u and r finite and making use of Eq. (29), we arrive at

$$\psi_{\mu\nu}^{\mathrm{elog}}(u, r, \theta) = -i(u + r)\psi_{\mu\nu}^{\tilde{m}_0}(u, r, \theta),$$

(38)

where

$$\psi_{\mu\nu}^{\tilde{m}_0}(u, r, \theta) \simeq e^{-i2\mu(u+r)-2i\theta} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -ir \\ 0 & -ir & -r^2 \end{pmatrix}.$$

(39)

It shows clearly that the massive wave solution (20) represents a critical massive graviton mode propagating in the Minkowski spacetime background.

On the other hand, we note here that the GCA generator M_0 acquires a rank-2 Jordan cell as

$$M_0 \begin{pmatrix} \psi_{\mu\nu}^{\mathrm{elog}} \\ \psi_{\mu\nu}^{\tilde{m}_0} \end{pmatrix} = \begin{pmatrix} 2\mu & 1 \\ 0 & 2\mu \end{pmatrix} \begin{pmatrix} \psi_{\mu\nu}^{\mathrm{elog}} \\ \psi_{\mu\nu}^{\tilde{m}_0} \end{pmatrix},$$

(40)

where 2μ is identified with the scaling dimension Δ.

IV. IS LOGARITHMIC GCA POSSIBLE?

As was shown previously, the BMS/GCA correspondence for the GMG provides the GCA_2 with the central charges $c_1 = 24k$ and $c_2 = 0$. What is the corresponding GCA to the critical GMG? It seems to be existed because of the presence of the rank-2 Jordan cell (40). In order to answer this question, we review the exotic rank-2 LCFT on the AdS_3 spacetime. The exotic rank-2 LCFT is composed of two operators $\{\mathcal{O}(z), \mathcal{O}^{\log}(z)\}$. The two-point functions of these operators take the forms [10]

$$< \mathcal{O}^1(z, \bar{z})\mathcal{O}^1(0, 0) > \ = \ 0, \tag{41}$$

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^1(0, 0) > \ = \ \frac{B}{2z^{2h(\tilde{m}_0)}\bar{z}^{2\bar{h}(\tilde{m}_0)}}, \tag{42}$$

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^{\log}(0, 0) > \ = \ -\frac{B\ln(m_L|z|^2)}{2z^{2h(\tilde{m}_0)}\bar{z}^{2\bar{h}(\tilde{m}_0)}}, \tag{43}$$

where B and m_L are non-zero parameters. We note that $< \mathcal{O}^1(z, \bar{z})\mathcal{O}^1(0, 0) >$ is null $(c_{\tilde{m}_0} = 0)$ for a LCFT. Here we use the relations

$$(h, \bar{h}) = \left(\frac{3}{2} + \frac{\ell\tilde{m}_0}{2}, \ -\frac{1}{2} + \frac{\ell\tilde{m}_0}{2}\right). \tag{44}$$

In order to match the flat-spacetime limit, we introduce two coordinates

$$z = t + \frac{1}{\ell}\phi, \ \ \bar{z} = t - \frac{1}{\ell}\phi \tag{45}$$

with $t = u + r$. In the flat-spacetime limit of $\ell \to \infty$, we recall that two expression of $h - \bar{h} = 2$ and $\lim_{\ell\to\infty}(h + \bar{h})/\ell = \Delta$ in Eq. (29) are finite. In the non-relativistic limit of $(z, \bar{z} \to \tau, \ \phi \to 0)$ induced by the flat-spacetime limit of $\ell \to \infty$ [14], the relations (42) and (43) reduce to

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^1(0, 0) > \ \sim \ \frac{1}{2t^{\infty}} \sim 0, \tag{46}$$

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^{\log}(0, 0) > \ \sim \ -\frac{\ln(t)}{t^{\infty}} \sim 0 \tag{47}$$

in the semi-infinite spacetime with a boundary condition at $\phi = 0$. For $t \neq 0$, these are null. This implies that the flat spacetime limit of the exotic rank-2 LCFT is not properly defined.

For a massive scalar Φ [14, 15], its corresponding two-point function of GCA_2 is given by

$$< \mathcal{O}(t)\mathcal{O}(0) > \sim \frac{1}{t^{2\Delta}}, \tag{48}$$

with $\Delta = \sqrt{m^2 + 1} + 1/2$.

Finally, we investigate the flat-spacetime limit of the standard rank-2 LCFT which corresponds to the critical case of $\tilde{m}_1 \ell = 1$. The non-vanishing two-point correlators are given by

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^L(0, 0) > \; = \; \frac{b_L}{2z^4}, \tag{49}$$

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^{\log}(0, 0) > \; = \; -\frac{b_L \ln(m_L|z|^2)}{z^4}, \tag{50}$$

where b_L and m_L are non-zero parameters and the zero left central charge, $c_L = 0$. Also, one has $h = 2$ and $\bar{h} = 0$. Choosing Eq. (45) and taking the non-relativistic limit, the relations (49) and (50) reduce to

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^L(0, 0) > \; \sim \; \frac{1}{2t^4}, \tag{51}$$

$$< \mathcal{O}^{\log}(z, \bar{z})\mathcal{O}^{\log}(0, 0) > \; \sim \; -\frac{\ln(t)}{t^4} \tag{52}$$

in the semi-infinite spacetime with a boundary condition at $\phi = 0$. The relations (51) and (52) have been dubbed a logarithmic GCA$_2$ as the non-relativistic limit of rank-2 LCFT [16–18]. However, there is a difference between logarithmic Schrödinger invariance and logarithmic Galilean conformal invariance [19]. In this case, two-point correlation functions are represented by a matrix representation

$$< \mathcal{O}^i\mathcal{O}^j > \sim \begin{pmatrix} 0 & \mathrm{CFT} \\ \mathrm{CFT} & \mathrm{Log} \end{pmatrix}, \tag{53}$$

where $i, j = L, \log$, CFT denotes the CFT two-point function (51), and Log represents (52). In this case, the critical wave solution to $D^2 h^{\log}_{\mu\nu} = 0$ is given by $h^{\log}_{\mu\nu} = -ith^\xi_{\mu\nu}$.

V. DISCUSSIONS

We have obtained the massive logarithmic wave solution from the critical GMG in three dimensional flat spacetime. In contrast to the logarithmic wave solution on the AdS$_3$ spacetime, it does not contain a logarithmic term in the flat spacetime.

At the off-critical point of GMG, the dual CFT is given by the GCA$_2$ with central charges $c_1 = 24k$ and $c_2 = 0$, where the mass parameter m disappears in the flat-spacetime limit.

On the other hand, at the critical point of GMG, we have discussed the logarithmic extension of the GCA which may arise from the exotic rank-2 LCFT on the boundary of the AdS_3 spacetime. The "exotic" rank-2 LCFT implies that it appears when two masses degenerate on the boundary of the AdS_3 spacetime. It turned out that the logarithmic GCA_2 is not realized from the exotic rank-2 LCFT. However, the logarithmic GCA_2 could be realized from the standard rank-2 LCFT when taking the flat spacetime limit.

Acknowledgments

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